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Eigenvalue multiplicity and energy of distance and Seidel matrices of cographs

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Abstract

Let $G = (V(G), E(G))$ be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The *distance* between the vertices $u, v \in V(G)$, denoted by $d_G(u, v)$, is the length of a shortest path between them in G, and define $d_G(u, u) = 0$ for all $u \in V(G)$. The *adjacency matrix* of a graph G on n vertices, denoted by $A(G)$, is the $n \times n$ matrix whose rows and columns are indexed by the vertex set of G and the (u, v)-th entry $A(G)_{uv} = 1$, if the vertices u and v are adjacent and $A(G)_{uv} = 0$ otherwise. The *Seidel matrix* of G is defined as $S(G) = J - I - 2A(G)$, where J and I are the all one's matrix and the identity matrix of order n, respectively. The *distance matrix* $D(G)$ of G is the $n \times n$ matrix with its rows and columns indexed by the vertices of G, and the (u, v)-th entry is equal to $d_G(u, v)$. The *distance energy* (respectively, *Seidel energy*) of G is the sum of the absolute values of all eigenvalues of $D(G)$ $\big($respectively, $S(G)\big)$. Two graphs are said to be *distance equienergetic* (respectively, *Seidel equienergetic*) if they have the same distance energy (respectively, Seidel energy).

A cograph is a graph with no induced path on four vertices. Recently, Lou and Lin in [Linear Algebra Appl. 608:1-12, 2021] proved that the multiplicity of any distance eigenvalue except -2 and -1 of a connected cograph G is at most the Dilworth number of G and gave example of an infinite family of cographs attaining this upper bound but this example is not correct.

In this article, we provide an infinite family of cographs for which the multiplicity of any distance eigenvalue except -2 and -1 is equal to the Dilworth number of the cographs. Moreover, we determine the multiplicities of the eigenvalues -2 and -1 of the distance matrix of a connected cograph G and construct a pair of non-cospectral distance equienergetic cographs for every $n \geq 6$. Very recently, in [Linear Algebra Appl. 698:56-72, 2024], the authors obtained an upper bound for any Seidel eigenvalue of a connected cograph other than -1 and 1. In this article, we find the multiplicities of the Seidel eigenvalues -1 and 1 of a connected cograph G and construct a pair of non-cospectral Seidel equienergetic cographs for every $n \geq 3$. We also give a construction for a pair of non-isomorphic Seidel cospectral cographs for every $n \geq 4$.

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